

LIFTS OF SIMPLE CURVES IN FINITE REGULAR COVERINGS OF CLOSED SURFACES

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ABSTRACT. Suppose S is a closed orientable surface and \tilde{S} is a finite sheeted regular cover of S . The following question was posed by Julién Marché in Mathoverflow: Do the lifts of simple curves from S generate $H_1(\tilde{S}, \mathbb{Z})$? A family of examples is given for which the answer is “no”.

1. INTRODUCTION

In this paper, the submodule of integral first homology generated by connected components of pre-images of simple closed curves is studied. More specifically, let S be a closed, orientable surface with base point, empty boundary and genus g . For $p: \tilde{S} \rightarrow S$ a finite sheeted regular covering of degree n , we say that the *simple curve homology relative to the cover p* (denoted by $sc_p(H_1(\tilde{S}; \mathbb{Z}))$) is the span of $[\tilde{\gamma}]$ in $H_1(\tilde{S}; \mathbb{Z})$ such that $\tilde{\gamma}$ is a connected component of $p^{-1}(\gamma)$ and γ a simple closed curve in S .

The *Torelli group* is the subgroup of the mapping class group that acts trivially on homology. Many deep questions about the mapping class group and from low dimensional geometry reduce to questions about the Torelli group. A survey can be found in [4]. Broadly speaking, one of the reasons many standard mapping class group techniques do not apply to the Torelli group is that homology classes can not be understood locally; they do not project to subsurfaces nor lift to covering spaces in any intuitive way. Some consequences of this are explained in [9]. The following question was posed by Julién Marché on Mathoverflow, [8], and arose while studying the ergodicity of the Torelli group on character varieties, [3]:

Question 1 (see [8]). (i) Does $sc_p(H_1(\tilde{S}; \mathbb{Z})) = H_1(\tilde{S}, \mathbb{Z})$?
(ii) If not, how can we characterize the submodule $sc_p(H_1(\tilde{S}; \mathbb{Z}))$?

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Since writing the first version of this paper, there has been much progress on answering Question 1 and related questions. The existence of counterexamples to part (i) of Question 1 was shown independently in [5]. In [5], existence of counterexamples follows indirectly from a construction of particular linear representations of the fundamental group.

A more subtle question arises when replacing \mathbb{Z} with \mathbb{Q} in Question 1. Let $Mod_{g,n}^p$ be the mapping class group of a surface with genus $g \geq 3$, p punctures and n boundary components, and Γ a finite index subgroup of $Mod_{g,n}^p$. The Ivanov conjecture states that $H_1(\Gamma; \mathbb{Q}) = 0$. Suppose Γ is now the subgroup of the mapping class group of \tilde{S} coming from lifts of mapping classes of S . An observation of Boggi-Looijenga, [7] is that when the submodule of $H_1(\tilde{S}; \mathbb{Q})$ spanned by lifts of simple curves has the same rank as $H_1(\tilde{S}; \mathbb{Q})$, the submodule of $H_1(\tilde{S}; \mathbb{Q})$ fixed by Γ is trivial. Theorem C of [10] then implies the Ivanov conjecture.

For free groups, a recent paper [2] showed that this and related questions are equivalent to statements in representation theory, also providing a framework in which to answer part (ii) of Question 1 and its analogues. The version of Question 1 with rational coefficients is however currently still open. A further discussion to the background of this question with integer and rational coefficients can be found in Section 8 of [2].

The intuition that lifts of simple curves should generate homology possibly stems from the fact that counterexamples necessarily have huge genus; “small” genus coverings can be shown to satisfy a plethora of conditions that guarantee this. For example, it seems to be general knowledge¹ that when the deck transformation group is Abelian, $sc_p(H_1(\tilde{S}; \mathbb{Z})) = H_1(\tilde{S}, \mathbb{Z})$.

Organisation of Paper. After providing some background and notation in Section 2, Section 3 studies a family of covering spaces in detail. Subsection 3.1 explains how to obtain spanning sets for homology of covers using relations in the deck transformation group. These results and examples are used in Subsection 3.2 to highlight differences between results for integral and rational homology, and in Subsection

¹As was explained to the author by Marco Boggi, [1], this claim follows from arguments of Boggi-Looijenga and [6] with rational and hence integral coefficients. It was proven directly for integral coefficients in an earlier version of this paper. For free groups and complex coefficients, this claim is Proposition 3.1 of [2].

3.3 to construct examples for which $sc_p(H_1(\tilde{S}; \mathbb{Z}))$ is a proper submodule of $H_1(\tilde{S}; \mathbb{Z})$.

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2. ASSUMPTIONS AND BACKGROUND

As pointed out by Ian Agol in [8], $sc_p(H_1(\tilde{S}; \mathbb{Z}))$ always coincides with $H_1(\tilde{S}; \mathbb{Z})$ when S is the torus. The 2-sphere has trivial first homology. Therefore we will only consider surfaces of genus at least three throughout the rest of this paper. Recall that the surface S is also assumed to be closed and oriented.

Curves and intersection numbers. A *curve* in S denotes the free homotopy class of the image of a smooth immersion of S^1 into S . In this convention, a curve is necessarily closed and connected. Also, a curve γ is said to be *simple* if the free homotopy class contains an embedding of S^1 in S . For the remainder of the paper, we will suppress the immersion and discussion of free homotopy class unless either is expressly needed and will often just select a particular immersion in the free homotopy class to work with. The algebraic intersection number of two curves, a and b , on the surface S is denoted by $\hat{i}_S(a, b)$, and the geometric intersection number, i.e. the minimum possible number of crossings between two representatives of the free homotopy classes a and b , is denoted by $i_S(a, b)$.

When it is necessary to work with based curves, the assumption will often be made that wherever necessary, a representative of the free homotopy class is conjugated by an arc, to obtain a curve passing through

the base point.

A symbol representing a curve with square brackets around it is used to denote the integer homology class with that curve as representative.

***d*-lifts.** Given an n -sheeted regular covering $p: \tilde{S} \rightarrow S$, the deck transformation group is denoted by D . An element $\gamma \in \pi_1(S)$ *d-lifts* to \tilde{S} if d is the order of the image of γ in D , i.e. $\gamma^d \in \pi_1(\tilde{S}) \subset \pi_1(S)$ and γ^k is not in $\pi_1(\tilde{S})$ for $0 < k < d$.

Primitivity. A homology class h is *primitive* if it is nontrivial and there does not exist an integer $k > 1$ and a homology class h_{prim} such that $h = kh_{\text{prim}}$.

3. EXAMPLES

We begin this section with a detailed study of a family of examples. Subsection 3.1 explains how to use relations in deck transformation groups to obtain spanning sets for the homology of covers. In Subsection 3.2, some properties of these covers are established that highlight the differences between integer and rational homology in this context. These results are then used in Subsection 3.3 to construct families of examples $p: \tilde{S} \rightarrow S$ for which $sc_p(H_1(\tilde{S}; \mathbb{Z}))$ is properly contained in $H_1(\tilde{S}; \mathbb{Z})$.

Consider a fixed presentation of

$$\pi_1(S) = \langle a_1, b_1, \dots, a_g, b_g | \Pi_{i=1}^g [a_i, b_i] \rangle$$

where a_i, b_i are curves representing a usual basis for $H_1(S; \mathbb{Z})$, satisfying $i(a_i, a_j) = i(b_i, b_j) = 0$ and $i(a_i, b_j) = \delta_j^i$.

For a prime number m , let ϕ be a homomorphism $\phi: \pi_1(S) \rightarrow (\mathbb{Z}/m\mathbb{Z})^{2g}$ such that

$$(1) \quad \begin{aligned} a_1 &\mapsto (1, 0, 0, \dots, 0) \\ b_1 &\mapsto (0, 1, 0, \dots, 0) \\ &\vdots \\ b_g &\mapsto (0, 0, 0, \dots, 1) \end{aligned}$$

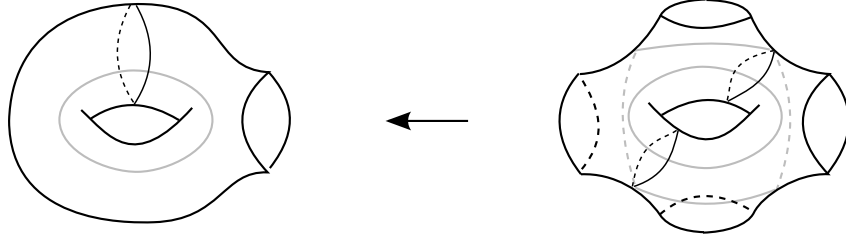


FIGURE 1. What happens to a 1-holed torus under the covering

Let $p : \tilde{S} \rightarrow S$ be the covering space corresponding to the kernel of ϕ , so that the deck transformation group is $D \cong \mathbb{Z}_m^{2g}$.

The cover of the genus two surface with $m = 2$ can be understood locally as a 4-fold cover of a once-punctured torus, as shown in Figure 1.

Each once-punctured torus lifts to four disjoint copies of 4-punctured tori, as shown in Figure 2. In turn, these disjoint copies are connected such that collapsing all of the punctured tori to points would result in the bipartite graph $K_{4,4}$, i.e. the bipartite graph with 8 vertices each of degree 4. The bipartite graph is drawn in grey in Figure 2.

The genus of \tilde{S} is 17 (for $g = 2$ and general m the genus is $m^4 + 1$) and so $H_1(\tilde{S}; \mathbb{Z})$ has rank 34.

This subsection is now concluded with an important observation from the example that will be needed later.

Lemma 2. *Let $\tilde{S} \rightarrow S$ be the cover from Equation 1. All simple, null homologous curves in S 1-lift to nonseparating curves in \tilde{S} .*

Proof. To start off with, the fact that null homologous curves 1-lift is a consequence of the fact that the cover has Abelian deck transformation group.

In Figure 2, the black curves are lifts of simple null homologous curves from S . These black curves all 1-lift to non-separating curves in \tilde{S} . Since the covering is characteristic, this observation is true independently of the choice of basis, therefore, all simple null homologous curves 1-lift to simple, non-separating curves in \tilde{S} . An analogous argument shows this is also true for $m > 3$. \square

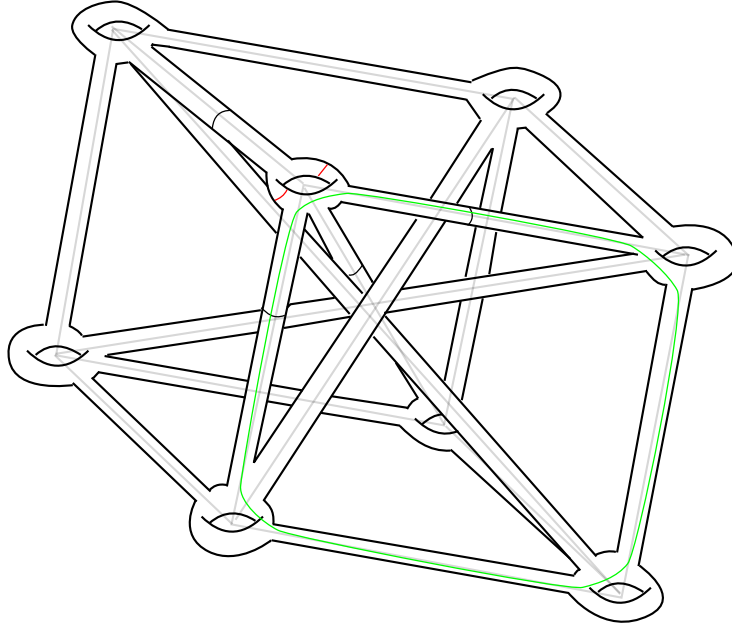


FIGURE 2. The covering space \tilde{S} . The red curves are some connected components of pre-images of the generator a_1 of $\pi_1(S)$. The green curve is a connected component of the lift of b_1b_2 , and the black curves are connected components of the lift of $[a_1, b_1]$.

3.1. Relations and spanning sets. It will now be shown how to use the relations of the deck transformation group to obtain a spanning set for $H_1(\tilde{S}, \mathbb{Z})$.

Suppose now that D is any group generated by no fewer than $2g$ elements, for which there is the short exact sequence

$$1 \rightarrow \pi_1(\tilde{S}) \rightarrow \pi_1(S) \xrightarrow{\phi} D \rightarrow 1$$

for some covering space \tilde{S} . Take $\{\phi(a_i), \phi(b_i)\}$ to be the generating set for D . Let $r := w(a_i, b_i)$ be a word in $\pi_1(S)$ that is mapped to the identity by ϕ ; in other words, $w(\phi(a_i), \phi(b_i)) = 1$ in D . The word r could be either nontrivial in $\pi_1(S)$, or it could be conjugate to the relation $\prod_i [a_i, b_i]$ in $\pi_1(S)$.

Lemma 3. *Suppose D and ϕ are as above. Let $\{r_1, r_2, \dots, r_k\}$ be a set of words in $\pi_1(S)$ mapping to a complete set of defining relations for D . Then the set of homology classes of connected components of the pre-images of the curves representing the words r_1, r_2, \dots, r_k is a*

spanning set for $H_1(\tilde{S}; \mathbb{Z})$.

Proof. A relation in the deck transformation group D can be expressed as a word in $\{\phi(a_1), \phi(b_1), \dots, \phi(a_g), \phi(b_g)\}$. Each of the r_i represents an element of $\pi_1(\tilde{S})$. Conversely, since D can not be generated by fewer than $2g$ elements, any element of $\pi_1(\tilde{S})$ is not just mapped to the identity in D , but to a relation in the generators of the group D .

When the image of $\{r_1, r_2, \dots, r_k\}$ is a complete set of defining relations for D , it follows that any element of $\pi_1(\tilde{S})$ is a product of conjugates of elements of the set $\{r_1, r_2, \dots, r_k\}$. Let c be a loop representing the element r_i . The connected components of $p^{-1}(c)$ correspond to conjugates of r_i . Therefore, the connected components of the pre-images of the closed curves represented by the words $\{r_1, r_2, \dots, r_k\}$ are a spanning set for $H_1(\tilde{S}; \mathbb{Z})$. \square

To use Lemma 3, a complete set of relations for D is needed. To start off with, there are the relations $\phi^m(a_i) = I$ and $\phi^m(b_i) = I$. These relations correspond to the submodule of $H_1(\tilde{S}; \mathbb{Z})$ spanned by connected components of pre-images of the generators. In Figure 2 with $m = 2$, some examples are drawn in red. There are also the commutation relations and the relations stating that the remaining group elements have order m in the deck transformation group. When $m = 2$, the commutation relations are a consequence of the relations stating that all 16 group elements have order m . For example,

$$\begin{aligned} \phi(a_1)\phi(b_1) &= (\phi(a_1)\phi(b_1))^{-1} \text{ since } \phi(a_1)\phi(b_1) \text{ has order two} \\ &= \phi(b_1)^{-1}\phi(a_1)^{-1} \\ &= \phi(b_1)\phi(a_1) \text{ since } \phi(a_1) \text{ and } \phi(b_1) \text{ each have order two} \end{aligned}$$

It will be shown later that this is a peculiarity of $m = 2$; for $m > 2$, we also need commutation relations. For $m = 2$, the relations stating that all elements of the deck transformation group are of order two are a complete set of relations for D . By Lemma 3, this gives us a set of simple, nonseparating curves whose pre-images span $H_1(\tilde{S}; \mathbb{Z})$.

3.2. Conclusions drawn from the examples, and integral versus rational homology. Examples for which $sc_p(H_1(\tilde{S}; \mathbb{Z}))$ can not be all of $H_1(\tilde{S}; \mathbb{Z})$ will now be constructed by showing that for $m > 2$, connected components of pre-images of separating curves are needed

to span $H_1(\tilde{S}; \mathbb{Z})$. The promised examples are then obtained by taking the composition of two such covering spaces, using Lemma 2.

Lemma 4. *In the covering space $p : \tilde{S} \rightarrow S$ from Equation 1 with $m \geq 3$, connected components of pre-images of simple, nonseparating curves do not span $H_1(\tilde{S}; \mathbb{Z})$.*

Proof. The lemma will be proven for $m = 3$ and it is claimed that analogous arguments work for $m > 3$.

Suppose a connected component of $p^{-1}([a_1, b_1])$ is in the span of connected components of pre-images of simple, nonseparating curves. Then there is a surface $\phi : \mathcal{F} \rightarrow \tilde{S}$, such that $p \circ \phi(\mathcal{F})$ has boundary $[a_1, b_1]^{-1} \gamma_1^3 \gamma_2^3 \gamma_3^3 \dots \gamma_k^3$, where the γ_i are simple, nonseparating curves. In the group $\pi_1(S)$ there is therefore the relation

$$(2) \quad [a_1, b_1]^{-1} \gamma_1^3 \gamma_2^3 \gamma_3^3 \dots \gamma_k^3 \kappa = I$$

where κ is in the subgroup $[\pi_1(\tilde{S}), \pi_1(\tilde{S})]$ of $\pi_1(S)$.

The product $\gamma_1^3 \gamma_2^3 \gamma_3^3 \dots \gamma_k^3$ is null homologous in S , so for every generator a_i (respectively b_i) the sum of the indices in the product $\gamma_1^3 \gamma_2^3 \gamma_3^3 \dots \gamma_k^3$ must be zero. This implies that the γ_i can not all be generators. Assume otherwise: then for every a_i^3 (respectively b_i^3) there must be an a_i^{-3} (respectively b_i^3), in which case $\gamma_1^3 \gamma_2^3 \gamma_3^3 \dots \gamma_k^3$ would be in the commutator subgroup of $\pi_1(\tilde{S})$. Since Lemma 2 states that a connected component of $p^{-1}([a_1, b_1])$ is nonseparating, this is not possible.

Suppose for simplicity that γ_i can be written as abc , where a, b and c are generators of $\pi_1(S)$. Then

$$\begin{aligned} \gamma_i^3 &= abcabcabc \\ &= abca^2bc[(bc)^{-1}, a^{-1}]bc \text{ using } ba = ab[b^{-1}, a^{-1}] \\ &= a^3bc[(bc)^{-1}, a^{-2}]bc[(bc)^{-1}, a^{-1}]bc \\ &= a^3bc[(bc)^{-1}, a^{-2}](bc)^2[(bc)^{-1}, a^{-1}][[(bc)^{-1}, a^{-1}]^{-1}, bc] \\ &= a^3(bc)^3[(bc)^{-1}, a^{-2}][[(bc)^{-1}, a^{-2}]^{-1}, (bc)^{-2}][(bc)^{-1}, a^{-1}] \\ &\quad [[(bc)^{-1}, a^{-1}]^{-1}, (bc)^{-1}] \\ &= a^3(bc)^3[(bc)^{-1}, a^{-2}]^{(bc)^{-2}}[(bc)^{-1}, a^{-1}]^{(bc)^{-1}} \end{aligned}$$

where for $x, y \in \pi_1(S)$, $x^y := yxy^{-1}$.

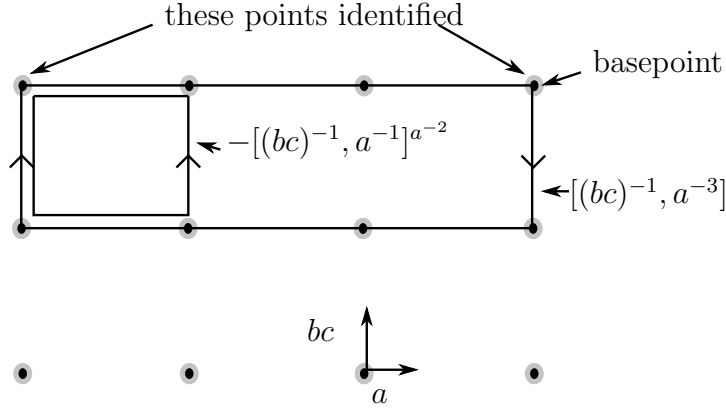


FIGURE 3. The dots represent elements of the fiber of the covering space $p : \tilde{S} \rightarrow S$. Translations in the vertical direction represent elements of the deck transformation corresponding to the lift of bc , and translations in the horizontal direction represent elements of the deck transformation group corresponding to a .

The product

$$A := [(bc)^{-1}, a^{-2}]^{(bc)^{-2}} [(bc)^{-1}, a^{-1}]^{(bc)^{-1}}$$

can be identified with an element of $\pi_1(\tilde{S})$. What is the corresponding homology class in $H_1(\tilde{S}; \mathbb{Z})$?

Supposing a choice of base point in \tilde{S} , as shown in Figure 3, $[(bc)^{-1}, a^{-2}]$ is homologous to

$$[(bc)^{-1}, a^{-3}] - [(bc)^{-1}, a^{-1}]^{a^{-2}}$$

It follows that A is homologous to

$$[[(bc)^{-1}, a^{-1}], a^{-2} (bc)^{-1}]^{(bc)^{-1}} + [(bc)^{-1}, a^{-3}]$$

As an element of $\pi_1(\tilde{S})$, A can therefore be written as the product of

$$[[(bc)^{-1}, a^{-1}], a^{-2} (bc)^{-1}]^{(bc)^{-1}}, [(bc)^{-1}, a^{-3}]$$

and an element of $[\pi_1(\tilde{S}), \pi_1(\tilde{S})]$.

In the product $\gamma_1^3 \gamma_2^3 \gamma_3^3 \dots \gamma_k^3$, repeat this argument on any γ_i of word length greater than one, to write $\gamma_1^3 \gamma_2^3 \gamma_3^3 \dots \gamma_k^3$ as a product of cubes of generators and terms such as A . Rearranging the order of terms that 1-lift to \tilde{S} , such as commutators and cubes of generators, only introduces terms in $[\pi_1(\tilde{S}), \pi_1(\tilde{S})]$. Also, the cubes of generators multiply to give

an element of $[\pi_1(\tilde{S}), \pi_1(\tilde{S})]$. Denote by w a word in $\pi_1(S)$. It follows that if the relation from Equation 2 is possible, a connected component of the pre-image of $[a_1, b_1]$ must be in the homological span of connected components of pre-images of words of the form:

$$(3) \quad \begin{aligned} &[w, a_i^{\pm 3}], [w, b_i^{\pm 3}] \text{ for } i \in 1, \dots, g, \text{ and} \\ &[w, [\pi_1(S), \pi_1(S)]] \end{aligned}$$

It follows from the commutator identities $[x, yz] = [x, y][x, z]^y$ and $[zx, y] = [x, y]^z[z, y]$ that elements of $[\pi_1(\tilde{S}), \pi_1(\tilde{S})]$ are also normally generated by words from Equation 3. Therefore $[a_1, b_1]$ must be in the subgroup N of $\pi_1(S)$ normally generated by words from Equation 3. We claim that this is not the case.

Let ψ be the homomorphism taking an element of $\pi_1(S)$ to its coset in $\pi_1(S)/N$, and ψ_1 be the homomorphism taking an element of $\pi_1(S)$ to its coset in $\pi_1(S)/[\pi_1(S), [\pi_1(S), \pi_1(S)]]$. We now compute the image of the commutator subgroup under ψ .

It follows from the commutator identities $[x, yz] = [x, y][x, z]^y$ and $[zx, y] = [x, y]^z[z, y]$ that $[\pi_1(S), \pi_1(S)]$ is generated by conjugates of commutators of generators. Since $[\pi_1(S), \pi_1(S)]$ maps to a subgroup in the center of $\psi_1(N)$, it follows that $\psi_1([\pi_1(S), \pi_1(S)])$ is generated by the image of commutators of generators of $\pi_1(S)$. The group $\psi_1([\pi_1(S), \pi_1(S)])$ is therefore a finitely generated, Abelian group.

Again using the commutator identities, it follows that $\psi_1([w, a_i^3]) = \psi_1([w, a_i]^3)$. Similarly for $\psi_1([w, b_i^3])$. If the word w is not a generator, it follows from the commutator identities and the fact that the image of the commutator subgroup under ψ_1 is in the center, that $\psi_1([w, a_i]^3)$ can be written as a product of cubes of commutators of generators. Since the word w can be taken to be any of the $2g$ generators, it follows that the image of $[\pi_1(S), \pi_1(S)]$ under ψ is a finitely generated Abelian group, each element of which has order three. In particular, $\psi([a_1, b_1])$ is not the identity. This proves the claim from which the lemma follows. \square

Remark 5. *The argument in Lemma 4 does not work when $m = 2$. This is because in this case we do not get any commutators of commutators in the expression for A , hence there is no contradiction to the existence of Equation 2.*

It follows from arguments of Boggi-Looijenga, [1] and [6], that when D is Abelian, $H_1(\tilde{S}; \mathbb{Q})$ is generated by homology classes of lifts of

simple, nonseparating curves. In Figure 1, for example, it is not hard to see that pairs of connected components of pre-images of a simple null homologous curve n are in the span of connected components of pre-images of generators. When $m > 3$, $m[\tilde{n}]$ is in the integral span of homology classes of connected components of lifts of simple, nonseparating curves. Hence $[\tilde{n}]$ is in the rational span, but not, as shown in Lemma 4, in the integral span.

3.3. Families of counterexamples. The promised families of examples for which lifts of simple curves do not span the integral homology of the covering space will now be constructed.

Let $\tilde{S} \rightarrow S$ be the covering with $m \geq 2$ just studied. Repeat the same construction, only with larger genus, and $m > 2$, on \tilde{S} to obtain a cover $\tilde{\tilde{S}} \rightarrow S$ factoring through \tilde{S} . That the result is a regular cover follows from the fact that it is a composition of two characteristic covers.

It is possible to see almost immediately that $sc_p(H_1(\tilde{\tilde{S}}, \mathbb{Z}))$ can not be all of $H_1(\tilde{\tilde{S}}, \mathbb{Z})$. In Lemma 4 we saw that 1-lifts of simple null homologous curves from \tilde{S} were needed to generate $H_1(\tilde{\tilde{S}}; \mathbb{Z})$. However, by Lemma 2, no simple null homologous curves in \tilde{S} project onto simple curves in S .

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